Median Eigenvalues of Subcubic Graphs

June 9. 2025

Joint with Hricha Acharya & Benjamin Jeter

Subcubic graphs: max deg ≤ 3.

Eigenvalues of adjacency matrix: $\lambda_1 \geq \cdots \geq \lambda_n$

Median eigenvalues: $\lambda_H = \lambda_{\lfloor \frac{n+1}{2} \rfloor}$. $\lambda_L = \lambda_{\lceil \frac{n+1}{2} \rceil}$.

Hückel Model Theory

Chemistry	Maths
Organic molecules	Chemical graphs (connected + subcubic)
π-electron energy levels	Eigenvalues
Highest occupied molecule obital energy	λ_H
Lowest un — — — —	λ_L
Molecule's kinetic stability	$\lambda_H - \lambda_L$

Fowler & Pisanski 2010

Computational experiments: most chemical graphs have med. eigenvals in $[-1, 1]$

Single exception: Heawood graph (3-reg. 14 vertices)

Incidence graph of Fano plane

Eigenvals: $-3, (-\sqrt{2})^6, (\sqrt{2})^6, 3$.

Conjecture: All but finitely many chemical graphs, $\lambda_H, \lambda_L \in [-1, 1]$.

Optimality of $[-1,1]$: Guo and Mohar constructed infinitely many bipartite chemical graphs with median eigenvals ± 1.

Known results

Fowler & Pisanski 2010: subcubic trees.

Mohar 2013: planar bipartite.

2016: bipartite except Heawood.

Wang & Zhang 2024 ..., Benediktovich 2014.

THM (Acharya, Jeter, J, 2025) All chemical but Heawood.

For simplicity, only focus on $\lambda_H \le 1$.

Proof of 99% of the cases:

(1) Take maximum cut of subcubic G, say (A, B)

(2). Assume in addition $|A| > |B|$

(3). Note max deg of $G[A] \le 1 \Rightarrow \lambda_1(G[A]) \le 1$

$\xRightarrow{\text{Cauchy interlace}} \lambda_H(G) \le \lambda_{1+|B|}(G) \le 1.$ $\square$

99% of the proof for the rest 1% of the cases

Key ideas:

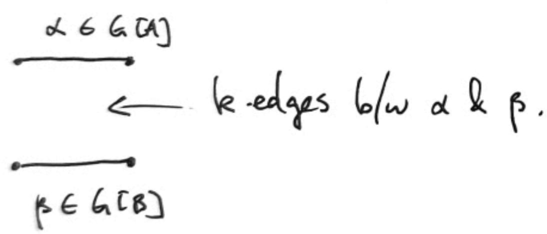
① Tail reducer: move k vertices, say C, from A to B s.t.

$\lambda_k(G[B \cup C]) \le 1$. (then Cauchy interlace implies $\lambda_{1} \le 1$).

② Cut enhancer: find C s.t. $(A \oplus C, B \oplus C)$ is larger cut, which is a contradiction.

③. Underlying graph (multigraph) M

vertices of M are edges of $G[A] \cup G[B]$. and

if 

then